# DAMAGE DETECTION BASED ON MODEL UPDATING METHODS

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# ABSTRACT

The paper treats the problem of detecting a structural damage with respect to location and extent from measured vibration test data. The method is based upon a mathematical model representing the undamaged vibrating structure and a local description of the damage, e.g. a finite element for a cracked beam.

A special chapter is devoted to the problem of modeling errors and their influence to damage localization accuracy. An approach is presented how to get reliable results also in this case.

The concept of inverse sensitivity equations is used which can be based on any type of data: e.g. modal data, FRFs, time series or a combination of them.

The resulting inverse problem usually is ill-posed, so that special attention must be paid to its accurate solution. The application to damage detection problems requires the reduction of a large set of damage parameter candidates to a small subset of one or two parameters really describing the local change of the system. An orthogonalization strategy is given to reduce the parameter set. The method is applied to laboratory structures in the frequency domain using FRFs and in the time domain. The results show that the algorithm is able to detect the damage.

#### **1. INTRODUCTION**

# 2. MATHEMATICAL MODEL OF THE SYSTEM

- 2.1 EQUATION OF MOTION
- 2.2 LOCAL MODELS FOR THE DAMAGE
- 2.3 DAMAGE SYMPTOMS AND SENSITIVITY

# 3. SOLUTION OF THE INVERSE PROBLEM

- 3.1 PARAMETER CHOICE BASED ON QR-DECOMPOSITION
- 3.2 OTHER SOLUTION METHODS

# 4. DAMAGE DETECTION WITH INACCURATE STRUCTURAL MODELS

- 4.1. REPLACEMENT OF MODEL DATA
- 4.2 APPLICATION OF STATE OBSERVERS

### 5. EXAMPLES

#### 5.1. DAMAGE IN A RECTANGULAR ALUMINUM PLATE

#### 5.1.1. System Description, Model and Measurement

As an application for the described damage detection procedure, a rectangular aluminum plate (size  $300 \times 600 \times 6 mm$ ) is investigated. Based on the measured weight, the density is calculated to  $\rho = 2800 kg / m^3$ . As a first guess of the Young's modulus  $E = 70\ 000\ MPa$  is used. The damage is represented by two orthogonal slots, each having a length of  $\approx 70\ mm$  and a width of  $\approx 4\ mm$ . The location of the damage and the 18 nodes where the system is excited by an impact hammer are shown in fig. 3. The acceleration pick-up is located at node no. 1. The 18 corresponding frequency response

functions (FRF) contain the first 10 modes in the measured frequency range. Free boundary conditions are obtained by fixing the structure with very soft springs. A comparison of the measured undamaged and damaged system is given in fig. 4. The shift of eigenfrequencies due to the damage are in a range of about 1 % to 2.5 %.

The FE model consists of flat shell elements (plate elements) with nine nodes and three dofs per node. Using a FE model discretized by  $3 \times 6 = 18$  elements the difference between measured and analytical eigenvalues for the undamaged case is minimized by global updating of the Young's modulus. This leads to a value of  $E \approx 67\,000\,MPa$  and a maximum difference in frequency of  $\approx 4.5\%$ . For a FE model discretized by  $9 \times 18$ elements (or even more) the remaining maximum difference of the eigenfrequencies after updating the Young's modulus is still  $\approx 3\%$ . Besides the fact of measurement errors, another explanation for these deviations is that the real plate is not exactly flat.

Despite of the shown inaccuracy and with respect to computing time the flat plate FE model consisting of 18 elements (fig. 3) shall be further used as original model. The corresponding eigenvalues and mode shapes are shown in fig. 5. The difference between the analytical  $(3 \times 6)$  and measured FRFs can be seen from fig. 6. It shall be noted that the deviations due to the incorrect model are of the same order as the deviations between damaged and undamaged system!

#### 5.2. DETECTION OF CRACKS IN A BEAM STRUCTURE

#### 5.2.1. Description of the system, Modelling and Measurement

This is an example for a localization in the time domain. Fig. 14 shows the T-like frame consisting of two welded straight aluminum bars under free-free boundary conditions realized by a soft spring. The system was excited by means of an impulse hammer at node 9 and the system responses were measured by means of 3 acceleration sensors at nodes no. 1 and 11 (y-direction both) and node no. 20 (x-direction).



Figure 14: Test structure and nodes of the FE model

The system is modelled by means of 19 finite Timoshenko beam elements and 3 lumped masses representing the sensors. The welded connection of the two beams must be considered by higher stiffnesses than the normal beam elements between the nodal points 5, 6, 7 and 12 (see fig. 14). To reduce calculation time, modal reduction is used. 20 modes (including the rigid body modes) were taken into account. Two sharp crack-like notches were introduced by means of wire erosion. The equations of motion remain linear because the cracks do not close (closing cracks are treated e.g. in [23,25]).

The crack model describes the relation between the stiffness reduction and crack depth a and crack position b in the element and is formulated as special finite beam element (fig. 15). The main idea [27-29] is based on principles of the linear-elastic fracture mechanics connecting the additional compliance of the element due to the damage via Castigliano's theorem to the decrease of elastic strain energy which is expressed in terms of the stress intensity factors [36].

- 5.2.2. Localization with an updated model
- 5.2.3. Damage localization from inaccurate models