DIRECT ESTIMATION OF NONLINEAR JOINT PARAMETERS

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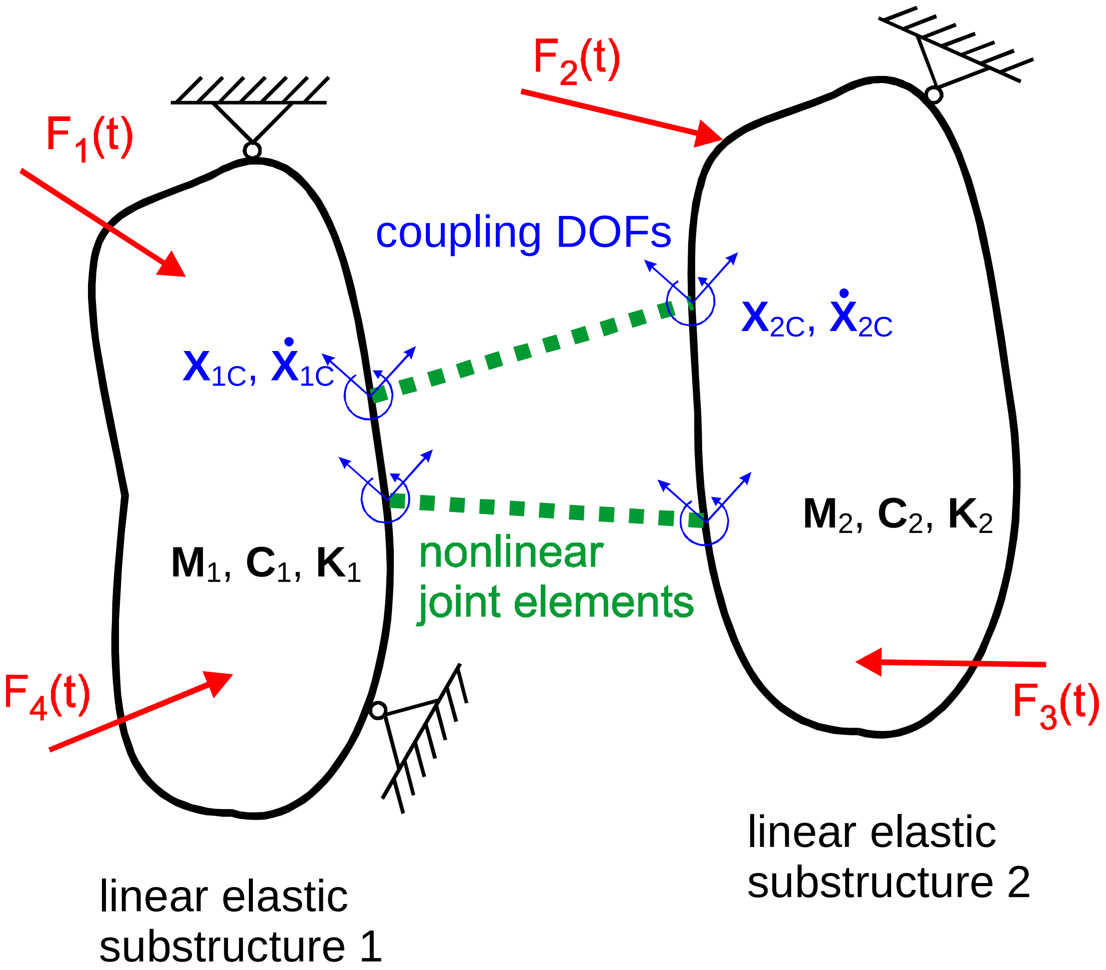
ABSTRACT

A method is proposed to identify nonlinear joint characteristics of a structure by means of measured data. The relation between the joint force function and the response of the system is established by use a position index matrix. The parameters of the joint force function are estimated by means of a least squares strategy. The parameter estimation can be done in time domain and also in frequency domain. Especially, when there is no excitation on the joint degrees of freedom only a part of the responses of the system are needed to identify the nonlinear joint parameters. The validity of the method is demonstrated by computer simulations.

Formulation of the problem



$$\begin{bmatrix} \mathbf{M}_{1,(n\times n)} & \\ & \mathbf{M}_{2,(m\times m)} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_1 \\ \ddot{\mathbf{X}}_2 \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{1,(n\times n)} & \\ & \mathbf{C}_{2,(m\times m)} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \end{Bmatrix} +$$

$$\begin{bmatrix} \mathbf{K}_{1,(n\times n)} & \\ & \mathbf{K}_{2,(m\times m)} \end{bmatrix} \begin{Bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{Bmatrix} +$$

$$+ \quad \mathbf{P}_{(n+m\times k)} \cdot \mathbf{g}(\mathbf{X}_{1c}, \mathbf{X}_{2c}, \dot{\mathbf{X}}_{1c}, \dot{\mathbf{X}}_{2c})_{(k\times 1)} = \mathbf{F}(t)_{(n+m\times 1)}$$

with **P** : position index matrix; each column a unit vector
g : nonlinear joint function

Common types of nonlinearities

with N_P, N_Q : orders of the nonlinearity

polynomial stiffness and damping link	$g_i(\mathbf{x}_c,\dot{\mathbf{x}}_c) = \sum_{l=1}^{N_P}\sum_{\substack{j=1\\ j\neq i}}^{N_e} c_{ijl}(\dot{x}_{ic}-\dot{x}_{jc})^l + \sum_{l=1}^{N_Q}\sum_{\substack{j=1\\ j\neq i}}^{N_e} k_{ijl}(x_{ic}-x_{jc})^l$
Cross-coupling nonlinearity link	$g_i(\mathbf{x}_c,\dot{\mathbf{x}}_c) = \sum_{l=0}^{N_P}\sum_{k=0}^{N_Q}\sum_{\substack{j=1\\ j\neq i}}^{N_e} \alpha_{ijlk}(x_{ic}-x_{jc})^l(\dot{x}_{ic}-\dot{x}_{jc})^k$
Coulomb friction and square-low damping link	$g_i(\dot{\mathbf{x}}_c) = \sum_{\substack{j=1\\ j\neq i}}^{N_e} \frac{\dot{x}_{ic}-\dot{x}_{jc}}{\lvert \dot{x}_{ic}-\dot{x}_{jc}\rvert} + \sum_{l=1}^{N_P}\sum_{k=1}^{N_Q}\sum_{\substack{j=1\\ j\neq i}}^{N_e} \beta_{ijlk}(\dot{x}_{ic}-\dot{x}_{jc})^l \lvert \dot{x}_{ic}-\dot{x}_{jc}\rvert^k$
piecewise nonlinear link	

In general: $g_i(\mathbf{x}_c,\dot{\mathbf{x}}_c) = a_{i1}f_1(\mathbf{x}_c,\dot{\mathbf{x}}_c) + \cdots + a_{iN_0}f_{N_0}(\mathbf{x}_c,\dot{\mathbf{x}}_c) = \sum_{j=1}^{N_0}\left(a_{ij}f_j(\mathbf{x}_c,\dot{\mathbf{x}}_c)\right)$

Equation of motion

$$(\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X}) + \mathbf{P}\mathbf{g} = \mathbf{F}$$

With

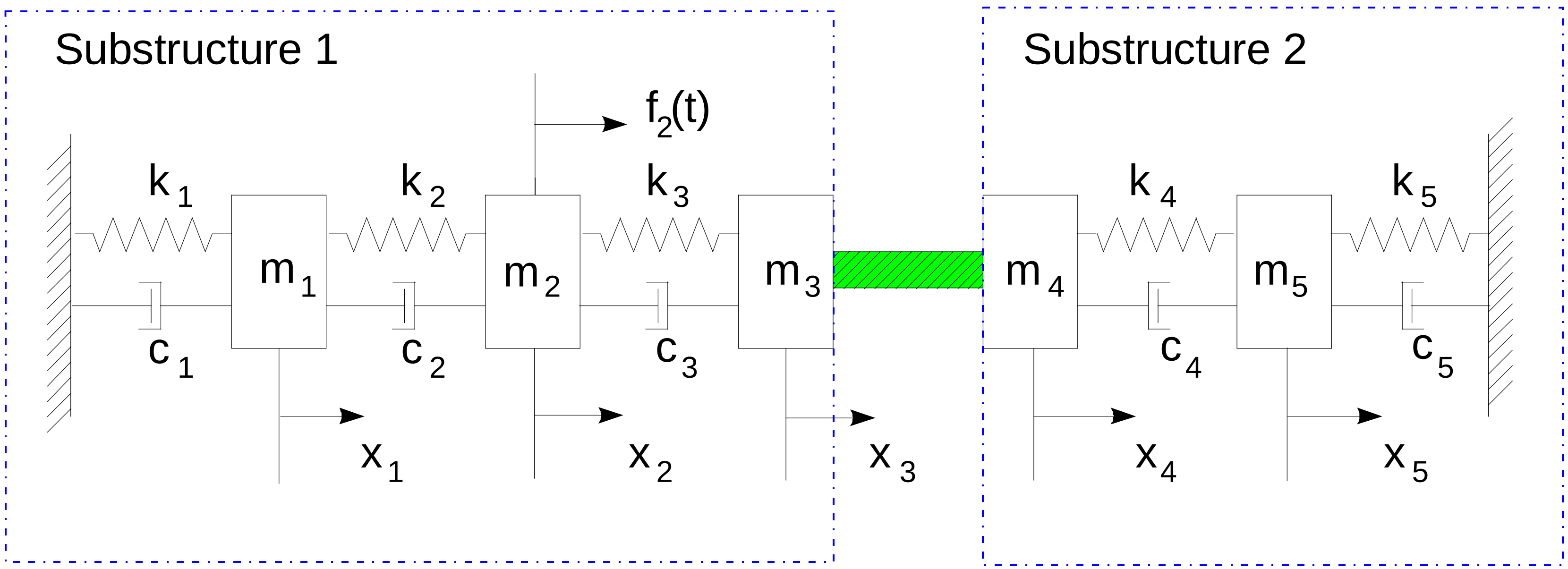
$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 & \\ & \mathbf{M}_2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_1 & \\ & \mathbf{C}_2 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_1 & \\ & \mathbf{K}_2 \end{bmatrix} \quad \mathbf{X} = [\mathbf{X}_1 \quad \mathbf{X}_2]^T$$

$$\Rightarrow \quad \mathbf{P}^T\mathbf{P}\mathbf{g} = \mathbf{P}^T\mathbf{F} - \mathbf{P}^T(\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X})$$

- where $\mathbf{P}^T\mathbf{P} = \mathbf{I}$
- Assumption: $\mathbf{F}(t)$ not acting on $\mathbf{X}_{1c}, \mathbf{X}_{2c}, \dot{\mathbf{X}}_{1c}, \dot{\mathbf{X}}_{2c}$

Example



Excitation $f_2(t) = 100 \sin(\omega t)$ $\omega = 50$ / sec

'measurement' $x_3, x_4, \dot{x}_3, \dot{x}_4$

Conclusions

- algorithm to identify the properties of nonlinear joint elements between linear elastic substructures
- only coupling DOF have to be measured
- assumption: no force on coupling DOF
- time domain and frequency domain identification
- identification for given nonlinear function type
- identification of the structure of the nonlinear function